In the Bubble Track Experiment, I calculated the kinetic energy and the momentum of an electron traveling through liquid hydrogen in the presence of a magnetic field, and compared those values to relativistic and classical models. I took measurements of the electron’s distance to the center along its spiraling path and its instantaneous radius. I used a map roller to measure the distance from the center along the spiral. To measure the instantaneous radius, I used a ruler and selected points about a centimeter away from the point. I then found the tangent lines of the curve at those three points, found the normal to those tangent lines, and used those three normal to approximate where the instantaneous center of the curve was for the point selected for measure the instantaneous radius. I then found the distance between my selected point and the instantaneous center: the instantaneous radius.

Classical Mechanics describes the relationship between momentum (), mass (), and velocity () as

(1)

Furthermore, the relationship between kinetic energy (), mass, and velocity as described by Classical Mechanics is

(2)

The equation

(3)

can be derived from Equations 1 and 2 by substituting in for in Equation 2. Equation 3 is how classical equations directly link kinetic energy and momentum for a single object of mass. The relativistic relationship between kinetic energy and momentum for a single object of mass is

(4)

where is the speed of light ( s-1)

In Figure 1, the red curve is a plot of Equation 3 and depicts what classical equations predict for the kinetic energy of an electron ( g) based on its momentum. The yellow curve is a plot of Equation 4 and depicts what the relativistic equation predicts for the kinetic energy of an electron based on its momentum. The blue data points are my calculations for the electron’s momentum and kinetic energy based on my measurements of the instantaneous radius and distance along the curve to the center of the spiral. The method of calculation can be found in the MATLAB code provided.

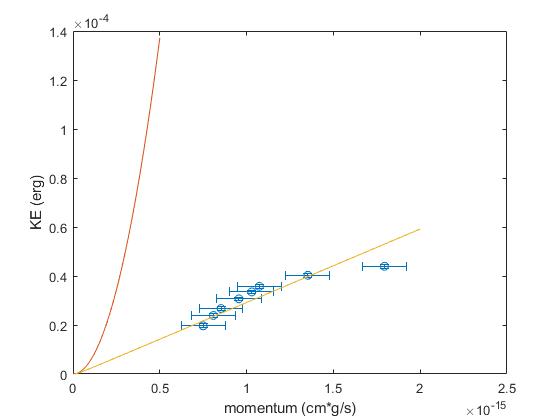


Figure 1: Theory curves for both classical (red) and relativistic (yellow) predictions of the electron’s kinetic energy based on its momentum plotted with measured data points and their uncertainties (blue).

In Figure 1, all of the data points are closer to the relativistic curve than they are to the classical curve, with two points falling on the line within both their momentum and kinetic energy uncertainty ranges. Because the data points are all closer to the values that the relativistic curve predicts than the values that the classical curve predicts, I conclude that the relativistic curve is more accurate.

To calculate the standard errors of the data I calculated the standard deviation for each of the eight points I took data from for both the radius and distance measurements. This involved taking the three measured values for each point, summing them, and then dividing the sum by the square root of the number of measurements, namely three. I then added in quadrature 0.85 cm for the radius measurements and 2.5 cm for the distance measurements to that quotient to produce the final uncertainty in my measurements for radius and distance. I added the measurement uncertainty to the statistical uncertainty because of the measuring devices I used (a roller and a ruler, with smallest measurement divisions of 1.0 cm and 0.1 cm, respectively), and because of the difficulty in measuring the instantaneous radius and distance along the spiral. The radius was difficult because drawing tangent and normal lines cannot be perfect, and the three normal lines did not always intersect at a single point. I estimate my measurements for the radius were off by 0.75cm because of this alone. Adding this to the uncertainty due to the smallest division of the ruler, I arrive at 0.85cm for measurement uncertainty for the radius. Additionally, the distance was difficult because rolling the map roller along the curve was tiring and I was not able to follow the center of the line perfectly all the time. I estimate my measurements for the distance were off by 1.5cm because of this alone. Adding this to the uncertainty due to the smallest division of the map roller, I arrive at 2.5cm for measurement uncertainty for the distance.

For the radius measurements the total uncertainty in the radius is

(5)

where is the total uncertainty in the radius, is the statistical uncertainty generated from calculating the standard error by finding the standard deviation, and 0.85 is the measurement uncertainty in centimeters. For distance measurements the total uncertainty in the distance is

(6)

where is the total uncertainty in the radius, is the statistical uncertainty generated from calculating the standard error by finding the standard deviation, and 2.5 is the measurement uncertainty in centimeters.

The uncertainties in the radius and in the distance of every point is shown in Table 1. The attached MATLAB code accepts this table as input, and converts the uncertainties in the radius and the distance into uncertainties in the momentum and kinetic energy, which can be seen as the error bars in Figure 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Point | Average Measured Radius (cm) | Uncertainty in Radius (cm) | Average Measured Distance (cm) | Uncertainty in Distance (cm) |
|  |  |  |  |  |
| 8 | 5.1 | 0.862 | 67 | 2.693 |
| 7 | 5.5 | 0.856 | 82 | 2.651 |
| 6 | 5.8 | 0.851 | 92 | 2.566 |
| 5 | 6.5 | 0.868 | 106 | 2.522 |
| 4 | 7 | 0.856 | 116 | 2.693 |
| 3 | 7.3 | 0.851 | 124 | 2.774 |
| 2 | 9.2 | 0.858 | 140 | 2.522 |
| 1 | 12.2 | 0.862 | 153 | 2.651 |

Table 1: A table showing the average measured radius, uncertainty in the average measured radius, the average measured distance, and the uncertainty in the average measured distance for each point.

These results show that the classical equation linking kinetic energy and momentum only holds for low momentum, low energy objects. Classical Mechanics does a poor job of modeling how much kinetic energy the electron has based on its momentum when an object of mass is going fast. Additionally, these results show that the relativistic equation linking kinetic energy and momentum is accurate for high energy, high momentum objects because my data points are much closer to what the relativistic equation predict for their values than what the classical equations predict for their values.

Knowing that the relativistic equations describe objects of mass that are moving very fast much better than classical equations do, I can use the equation

(7)

(where is momentum, , is the speed of light, is the speed of the object, and is the mass of the object) to calculate the speed of the electron at every point by taking the calculated value for the momentum of the electron at every point and solving for . The equation

(8)

Can be used to calculate the speed of the electron based on its mass (), the speed of light , and its calculated momentum . Table 2 shows the resulting speeds of the electron at every point based on its calculated momentum. As Table 2 shows, the electron slows down as it travels towards the center of its spiral. This is consistent with what one would expect to happen, because the electron is colliding with other particles and losing energy as it goes through the hydrogen in the bubble chamber.

|  |  |  |  |
| --- | --- | --- | --- |
| Point | Calculated Momentum (g cm s-1) | Calculated Speed (cm s-1) | Calculated Speed (cm s-1 c-1) |
| 8 | 7.49985600000000e-16 | 29959390648.17834 | 0.9993377034247586 |
| 7 | 8.08808000000000e-16 | 29962171274.197056 | 0.9994304551249603 |
| 6 | 8.52924800000000e-16 | 29963890601.77589 | 0.9994878057197787 |
| 5 | 9.55864000000000e-16 | 29967017877.07852 | 0.9995921203954544 |
| 4 | 1.02939200000000e-15 | 29968701447.269653 | 0.9996482782522051 |
| 3 | 1.07350880000000e-15 | 29969549886.57602 | 0.999676579141161 |
| 2 | 1.35291520000000e-15 | 29973140080.72694 | 0.9997963351275148 |
| 1 | 1.79408320000000e-15 | 29975773237.63264 | 0.9998841677875913 |

Table 2: A table showing the calculated relativistic momentum, calculated speed, and calculated speed as a fraction of .

Knowing that the electron is always moving at very close to the speed of light through all my measurements, I can calculate the theoretical slope of the kinetic energy vs momentum curve when a particle is moving at almost the speed of light by taking the derivative of the kinetic energy with respect to momentum and then taking the limit as the speed approaches .

The derivative of Equation 4 with respect to is

The limit as approaches is